

# A new empirical approach to quark and lepton masses

Kevin Loch

PO Box 222081 Chantilly, VA 20153\*

(Dated: 2017 February 27)

We present an alternative to the Koide formula and its extensions. By introducing two parameters  $k$  and  $\alpha_f$  derived from charged leptons we are able to construct new empirical formulas that appear to relate all fundamental fermion pole masses. The predicted masses are in excellent agreement with known experimental values and constraints for heavy quarks and neutrinos. For light quarks we predict speculative pole masses of the same order of magnitude as  $\mu = 1$  GeV  $\overline{\text{MS}}$  masses but higher by a factor of  $\sim 1.5$ . The condition where  $k^{12} = 3.5$  (exact) is also considered as it would allow ultra high precision predictions.

PACS numbers: 14.60.Pq, 12.15.Ff, 12.10.Kt

One outstanding problem in modern physics is the seemingly unrelated particle masses that exist as free parameters in the standard model. Numerous attempts have been made over the years to derive a systematic mathematical relationship between these masses, most famously by Yoshio Koide with his charged lepton formula [1, 2]

$$\frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} \simeq \frac{2}{3}. \quad (1)$$

While this approximation works for charged leptons, and appears to work for some combinations of quarks with running masses, it does not work for neutrinos or attempt to relate all fundamental fermions in a consistent manner. It does however hint at the possibility of finding other more fundamental relationships. Indeed, Koide and others [1–24] have used it as inspiration for extended models to cover other fermions.

While these extended Koide models are interesting our goal is to instead look for new empirical relationships in the latest quark mass and neutrino mixing data. After deriving two new parameters from charged leptons we find they enable an interesting set of formulas that appear to relate all fundamental lepton and quark pole (physical) masses. This allows precise predictions of neutrino and quark pole masses and if correct would significantly reduce the number of free parameters in the standard model (extended for neutrinos with mass).

Our first parameter comes from a charged lepton relationship that is close to unity:

$$\frac{m_e m_\tau^2}{m_\mu^3} = 1.36777(18) \simeq 3.5^{1/4}. \quad (2)$$

If we consider this dimensionless residual value to be something peculiar to the muon we can define a second generation normalization factor  $k$

$$k \equiv \frac{(m_e m_\tau^2)^{1/3}}{m_\mu}. \quad (3)$$

With the latest values for  $m_e$ ,  $m_\mu$  [25] and  $m_\tau$  [26] we find  $k = 1.110039(50) \simeq 3.5^{1/12}$ . This constant allows us to create a charged lepton family equation

$$\frac{m_e m_\tau^2}{(k m_\mu)^3} = 1. \quad (4)$$

Our second parameter is used to relate the widely different scales observed between light and heavy particles. Mac Gregor proposed using the QED coupling constant to explain fermion lifetimes and mass relationships with the zero energy scale  $\alpha_{\text{QED}}(0)$  [27, 28]. Instead of the actual QED constant we propose a fermion mass hierarchy parameter  $\alpha_f$  with a value similar to an effective QED coupling constant. In our search for suitable values we considered the range  $125 < \alpha_f^{-1} < \alpha_{\text{QED}}^{-1}(0)$ . While initially targeting  $\alpha_{\text{QED}}^{-1}(M_Z^2) = 128.944$  [29] we discovered the useful relationship

$$\alpha_f \equiv 27 \frac{m_e}{m_\tau}. \quad (5)$$

Using the latest values for  $m_e$  [25] and  $m_\tau$  [26] we find  $\alpha_f^{-1} = 128.7862 \pm 0.0087$ . Despite the similarity in value to an effective QED coupling constant we consider  $\alpha_f$  to be a separate parameter related only to mass generation and not electric charge. Regardless of the true physical nature of  $\alpha_f$ , we find that it along with  $k$  is useful for analyzing fermion mass relationships. With (4) and (5) we can create charged lepton equations

$$m_\mu = 9 \frac{m_e}{k \alpha_f^{2/3}}, \quad (6)$$

$$m_\tau = 27 \frac{m_e}{\alpha_f}. \quad (7)$$

With our choice of definitions for  $k$  and  $\alpha_f$  we have established an interesting and surprisingly simple charged lepton mass hierarchy that could be used to look for similar patterns in other sectors.

We will now use the parameters  $k$  and  $\alpha_f$  to look for similar empirical formulas for left handed neutrinos and

\* kevinmloch@gmail.com

predict mass state values for them. While neutrino mass states have not been directly measured yet numerous neutrino oscillation experiments have established increasingly refined limits on neutrino squared mass differences [30]. Cosmological models also put constraints on the sum of all three mass states [31]. The best fits we have found using  $k$ ,  $\alpha_f$  and simple coefficients to the experimental active neutrino squared mass differences are mass states

$$m_1 = \alpha_f^4 m_e = 1.85756 \pm 0.00050 \times 10^{-3} \text{ eV}/c^2, \quad (8)$$

$$m_2 = 9 \frac{\alpha_f^4}{k^6} m_e = 8.9363 \pm 0.0034 \times 10^{-3} \text{ eV}/c^2, \quad (9)$$

$$m_3 = 27 \alpha_f^4 m_e = 5.0154 \pm 0.0013 \times 10^{-2} \text{ eV}/c^2. \quad (10)$$

These predicted masses give squared mass differences of

$$\Delta m_{21}^2 = 7.6407 \pm 0.0061 \times 10^{-5} \text{ eV}^2/c^4, \quad (11)$$

$$\Delta m_{31}^2 = 2.5120 \pm 0.0013 \times 10^{-3} \text{ eV}^2/c^4, \quad (12)$$

$$\Delta m_{32}^2 = 2.4356 \pm 0.0013 \times 10^{-3} \text{ eV}^2/c^4, \quad (13)$$

and the sum of all three mass states

$$\Sigma m_\nu = 6.0948 \pm 0.0013 \times 10^{-2} \text{ eV}/c^2.$$

Mass ordering is normal with  $m_1 < m_2 < m_3$ . These are in excellent agreement with recent global analysis of oscillation experiments [26, 30] and just below cosmological model limits [31]. A comparison of our predicted values to experiments is shown in table I. Remarkably, these equations have the identical integer coefficients (1, 9, 27) as our charged lepton formulas and also satisfy a left handed neutrino family equation

$$\frac{m_1 m_3^2}{(k^6 m_2)^3} = 1, \quad (14)$$

similar to (4) for charged leptons. Right handed sterile neutrinos, if the exist do not yet have reliable constraints established from which we could search for similar relationships.

We will now attempt to apply the techniques we used relating masses in the lepton sector to quarks. While most attempts to find relationships between leptons and quarks use renormalized running masses we have surprisingly found the best overall results using pole masses. The deconfined nature of the t-quark has enabled It's mass to be measured through decay products in particle collision experiments. The best fit we have found using  $\alpha_f$  and simple coefficients to the experimental t-quark mass is

$$M_t = \frac{1}{2\pi\alpha_f^3} m_e = 173.719 \pm 0.035 \text{ GeV}/c^2. \quad (15)$$

For quarks we use the notation  $M_q$  to denote pole masses and  $m_q^{\overline{\text{MS}}}$  for renormalized reference masses. Several different analysis modes exists on data from the ongoing t-quark experiments at the LHC with a comprehensive summary in [32]. Our predicted pole mass is an excellent match for recent analysis including the 2016 PDG average of  $M_t = 173.21 \pm 0.51 \pm 0.71$  [26]. We also have a particularly close match to the center value of  $M_t = 173.7 \pm 1.5 \pm 1.4_{-0.5}^{+1.0}$  reported in [33].

For the b- and c-quarks we target the pole masses derived from theoretical  $\overline{\text{MS}}$  masses. The best fits we have found using  $k$ ,  $\alpha_f$  and simple coefficients for the b- and c-quarks are

$$M_b = \frac{1}{\sqrt{\pi}\alpha_f^2} m_e = 4.78171 \pm 0.00064 \text{ GeV}/c^2, \quad (16)$$

$$M_c = \frac{2}{\pi k^{11}\alpha_f^2} m_e = 1.71127 \pm 0.00088 \text{ GeV}/c^2. \quad (17)$$

These predicted pole masses are in excellent agreement with recent pole mass evaluations of  $M_b = 4.78 \pm 0.06 \text{ GeV}$  and  $M_c = 1.67 \pm 0.07 \text{ GeV}$  [26].

For the light quarks perturbative QCD does not provide reliable pole masses so we are left to extrapolate relationships from the heavy quarks. For the purpose of extrapolation we assume that two quark family equations exist similar to (4) and (14) for the leptons. This is straightforward for the u-quark using (15), (17) and (21). For the d- and s-quarks we have made our best attempt to find equations of similar form as the b- quark with resulting masses of the same order of magnitude but higher than the  $\mu = 1 \text{ GeV}$   $\overline{\text{MS}}$  mass. It must be emphasized that our light quark formulas are speculative and are only shown to demonstrate the ability to predict masses of expected magnitude while satisfying the proposed family equations. The best fits we have found using  $k$ ,  $\alpha_f$  and simple coefficients for the u-, d- and s-quarks are

$$M_u = \frac{32}{\pi} m_e = 5.204992524(32) \text{ MeV}/c^2, \quad (18)$$

$$M_d = \frac{27}{\sqrt{\pi}} m_e = 7.784107630(47) \text{ MeV}/c^2, \quad (19)$$

$$M_s = \frac{3}{\sqrt{\pi}k^{12}\alpha_f^{4/3}} m_e = 160.719 \pm 0.088 \text{ MeV}/c^2. \quad (20)$$

The quark family equations are then given by

$$\frac{M_u M_t^2}{(k^{11} M_c)^3} = 1, \quad (21)$$

$$\frac{M_d M_b^2}{(k^{12} M_s)^3} = 1. \quad (22)$$

These predicted pole masses are the same order of magnitude but higher than recent evaluations at  $\mu = 1 \text{ GeV}$   $\overline{\text{MS}}$ . We compare them to the Aoki [34]  $\mu = 2 \text{ GeV}$

$\overline{\text{MS}}$  masses scaled by a factor of 1.35 to  $m_u^{\overline{\text{MS}}}(1\text{GeV}) = 3.47$  MeV,  $m_d^{\overline{\text{MS}}}(1\text{GeV}) = 4.97$  MeV, and  $m_s^{\overline{\text{MS}}}(1\text{GeV}) = 112.86$  MeV. The PDG review [26] points out that u- and d-quark masses are not without controversy. We also see there is significant variance in results for the u-quark. Using the Aoki masses for light quarks scaled by 1.35 to 1GeV the ratio of our predicted pole light quark masses to the corresponding  $\mu = 1$  GeV  $\overline{\text{MS}}$  reference masses is  $\sim 1.5$  for the u- and d-quarks and 1.43 for the s-quark. Our predicted mass ratio  $M_u/M_d \simeq 0.669$  is higher than the 2016 PDG evaluation of 0.38-0.58 but consistent with

the Aoki result of  $0.698 \pm 0.051$  used in the PDG evaluation.

While the preceding formulas in terms of  $m_e$  are convenient for calculating precision predicted values it is more interesting to analyze them together in terms of the Higgs vacuum expectation value  $v \simeq 246.22$  GeV [26]. With the top quark Yukawa coupling  $y_t = \sqrt{2}M_t/v \simeq 0.99779$  we can rewrite all of our proposed formulas in terms of  $y_t$  and  $v$ :

$$\begin{aligned}
m_1 &= \sqrt{2}\pi\alpha_f^7 y_t v, & m_e &= \sqrt{2}\pi\alpha_f^3 y_t v, & M_d &= 27\sqrt{2}\pi\alpha_f^3 y_t v, & M_u &= \frac{64}{\sqrt{2}}\alpha_f^3 y_t v, \\
m_2 &= 9\sqrt{2}\pi\frac{\alpha_f^7}{k^6} y_t v, & m_\mu &= 9\sqrt{2}\pi\frac{\alpha_f^{7/3}}{k} y_t v, & M_s &= 3\sqrt{2}\pi\frac{\alpha_f^{5/3}}{k^{12}} y_t v, & M_c &= \frac{4}{\sqrt{2}}\frac{\alpha_f}{k^{11}} y_t v, \\
m_3 &= 27\sqrt{2}\pi\alpha_f^7 y_t v, & m_\tau &= 27\sqrt{2}\pi\alpha_f^2 y_t v, & M_b &= \sqrt{2}\pi\alpha_f y_t v, & M_t &= \frac{1}{\sqrt{2}} y_t v, \\
\frac{m_1 m_3^2}{(k^6 m_2)^3} &= 1, & \frac{m_e m_\tau^2}{(k m_\mu)^3} &= 1, & \frac{M_d M_b^2}{(k^{12} M_s)^3} &= 1, & \frac{M_u M_t^2}{(k^{11} M_c)^3} &= 1.
\end{aligned} \tag{23}$$

There are several interesting patterns in these expressions, only some of which are enforced by the family equations. Despite these patterns there does not appear to be any relationship between them and electric charge or any other properties besides generation, family and mass. The lack of obvious equations for each row relating all fermions in a specific generation leaves open the question of whether we are missing families such as right handed neutrinos or superpartners. It is however possible to write several combinations of the family equations that cancel factors of  $k$ , for example:

$$\frac{M_s^3 M_t^2 m_\tau^2 M_u m_e}{M_c^3 m_\mu^3 M_b^2 M_d} = 1, \tag{24}$$

$$\frac{m_2^{12} M_t^2 M_b^2 m_\tau^2 M_d M_u m_e}{m_3^8 m_1^4 M_c^3 M_s^3 m_\mu^3} = 1, \tag{25}$$

$$\frac{m_2^6 M_b^2 M_d}{m_3^4 m_1^2 M_s^3} = 1. \tag{26}$$

In table I we provide a summary of the predicted masses and parameters derived from these equations.

The precision of the predicted values in table I is limited by the relative uncertainty in the tau mass of

$6.8 \times 10^{-5}$  [26]. In (3) we note that  $k$  is approximately  $3.5^{1/12}$ . In fact  $k$  differs from  $3.5^{1/12}$  by a factor of only  $1.8 \times 10^{-6}$  which is significantly less than the relative uncertainty in the tau. If we assume that  $k^{12} = 3.5$  (exact) then

$$m_\tau = \left( \frac{(k m_\mu)^3}{m_e} \right)^{\frac{1}{2}} = 1776.864828(61) \text{ MeV}/c^2. \tag{27}$$

This would allow a higher precision value for  $\alpha_f$  and for the predicted masses. While there is no known reason for the significance of the number 3.5 to justify this assumption, we provide a list of the ultra high precision calculations in table II for reference.

In conclusion, we have found new empirical fermion mass relationships that are surprising in several ways. Instead of the conventional expectation that such relationships would only appear at higher energy scales using renormalized masses, these formulas appear to relate pole (physical) masses across all sectors. They relate individual fermion masses to each other across families while also satisfying an equation for each family. The patterns in the coefficients and exponents suggest that they could lead to a better understanding of the mass hierarchy. Finally, they predict masses in excellent agreement with known experimental values and constraints. However, as with any empirical formulas they could just be coin-

Parameter	THIS WORK	Reference Value	Ref.
$m_e$	Used as input	0.5109989461(31) MeV	CODATA 2014 [25]
$m_\mu$	Used as input	105.6583745(24) MeV	CODATA 2014 [25]
$m_\tau$	Used as input	1776.86 $\pm$ 0.12 MeV	PDG 2016 [26]
$k$	1.110039(50)		
$\alpha_f$	0.00776481(52)		
$\alpha_f^{-1}$	128.7862 $\pm$ 0.0087		
$m_1$	1.85756 $\pm$ 0.00050 $\times 10^{-3}$ eV		
$m_2$	8.9363 $\pm$ 0.0034 $\times 10^{-3}$ eV		
$m_3$	5.0154 $\pm$ 0.0013 $\times 10^{-2}$ eV		
$\Delta m_{21}^2$	7.6407 $\pm$ 0.0061 $\times 10^{-5}$ eV <sup>2</sup>	7.53 $\pm$ 0.18 $\times 10^{-5}$ eV <sup>2</sup>	PDG 2016 [26]
$\Delta m_{31}^2$	2.5120 $\pm$ 0.0013 $\times 10^{-3}$ eV <sup>2</sup>	2.524 <sup>+0.038</sup> <sub>-0.041</sub> $\times 10^{-3}$ eV <sup>2</sup>	Esteban 2016 [30]
$\Delta m_{32}^2$	2.4356 $\pm$ 0.0013 $\times 10^{-3}$ eV <sup>2</sup>	2.44 $\pm$ 0.06 $\times 10^{-3}$ eV <sup>2</sup>	PDG 2016 [26]
$\sum m_\nu$	6.0948 $\pm$ 0.0013 $\times 10^{-2}$ eV	< 9.68 $\times 10^{-2}$ eV	Giusarma 2016 [31]
$M_c$	1.71127 $\pm$ 0.00088 GeV	$M_c = 1.67 \pm 0.07$ GeV	PDG 2016 [26]
$M_b$	4.78171 $\pm$ 0.00064 GeV	$M_b = 4.78 \pm 0.06$ GeV	PDG 2016 [26]
$M_t$	173.719 $\pm$ 0.035 GeV	$M_t = 173.21 \pm 0.51 \pm 0.71$ GeV	PDG 2016 [26]
$y_t$	0.99779 $\pm$ 0.000020		
$M_u$	5.204992524(32) MeV	$m_u^{\overline{\text{MS}}}(1\text{GeV}) = 3.47 \pm 0.35$	Aoki 2012 [34]
$M_d$	7.784107630(47) MeV	$m_d^{\overline{\text{MS}}}(1\text{GeV}) = 4.97 \pm 0.39$	Aoki 2012 [34]
$M_s$	160.719 $\pm$ 0.088 MeV	$m_s^{\overline{\text{MS}}}(1\text{GeV}) = 112.86 \pm 0.78$	Aoki 2012 [34]
$M_u/M_d = 32/(27\sqrt{\pi})$	0.66866913...	0.698 $\pm$ 0.051	Aoki 2012 [34]
$M_u/m_u^{\overline{\text{MS}}}(1\text{GeV})$	1.50 $\pm$ 0.17		
$M_d/m_d^{\overline{\text{MS}}}(1\text{GeV})$	1.57 $\pm$ 0.12		
$M_s/m_s^{\overline{\text{MS}}}(1\text{GeV})$	1.4241 $\pm$ 0.0099		

Table I. Predicted pole masses and derived values. Light quark  $\mu = 2 \text{ GeV } \overline{\text{MS}}$  reference masses have been rescaled to  $\mu = 1 \text{ GeV}$  by multiplying by 1.35.

cidental or approximations of more complex underlying phenomena. The most exciting area of experimental confirmation would be refinement of the neutrino oscillation parameters and especially measurement of the neutrino mass states directly. The KATRIN experiment [35] will search for neutrino mass signatures in decay from tritium  $\beta$ -decay but has a minimum sensitivity threshold

of 0.2 eV, 100 times higher than our predicted  $m_1$  and 25 times higher than our predicted  $m_3$ . Further refinement of the t-quark mass is another important confirmation opportunity. Further refinement of the Tau mass could provide support for the  $k^{12} = 3.5$  (exact) condition which would allow ultra high precision predictions of other masses as shown in Table II.

[1] Y. Koide, Nuovo Cim. Lett. 34, 201 (1982).

[2] Y. Koide, A Fermion-Boson Composite Model of quarks and leptons, Phys. Lett. B 120 (1983) 161.

[3] Y. Koide, Phys. Rev. D 28 (1983) 252.

[4] R. Foot (1994), A note on Koide's lepton mass relation, arXiv:hep-ph/9402242.

[5] Y. Koide and M. Tanimoto, U(3)-Family Nonet Higgs Boson and its Phenomenology, Z.Phys.C72:333-344,1996, doi:10.1007/s002880050253, arXiv:hep-ph/9505333.

[6] Y. Koide and H. Fusaoka, Analytical Expressions of Masses and Mixings in a Democratic Seesaw Mass Matrix Model, Prog.Theor.Phys.97:459-478,1997, doi:10.1143/PTP.97.459, arXiv:hep-ph/9612322.

[7] Nan Li, Bo-Qiang Ma, Estimate of neutrino masses from Koide's relation, Phys.Lett. B609 (2005) 309-316, arXiv:hep-ph/0505028.

[8] A. Rivero, A. Gsponer, The strange formula of Dr. Koide, arXiv:hep-ph/0505220. doi:10.1016/j.physletb.2005.01.066,

[9] W. Krolkowski (2005), Excellent approximate solution to the mysterious mass equation by Koide, arXiv:hep-ph/0508039.

[10] J. M. Gerard, F. Goffinet, and M. Herquet, A New Look at an Old Mass Relation, Phys.Lett. B633 (2006) 563-566, doi:10.1016/j.physletb.2005.12.054, arXiv:hep-ph/0510289

Parameter	THIS WORK if $k^{12} = 3.5$ (exact)
$k^{12}$	3.5 (exact)
$k$	1.110040958331563...
$\alpha_f$	0.00776478398(27)
$\alpha_f^{-1}$	128.7865835(45)
$m_\tau$	1776.864828(61) MeV
$m_1$	$1.85753718(26) \times 10^{-3}$ eV
$m_2$	$8.9360585(12) \times 10^{-3}$ eV
$m_3$	$5.01535040(70) \times 10^{-2}$ eV
$\Delta m_{21}^2$	$7.6402697(21) \times 10^{-5}$ eV <sup>2</sup>
$\Delta m_{31}^2$	$2.5119235(70) \times 10^{-3}$ eV <sup>2</sup>
$\Delta m_{32}^2$	$2.4355208(70) \times 10^{-3}$ eV <sup>2</sup>
$\sum m_\nu$	$6.0947100(70) \times 10^{-2}$ eV
$M_c$	1.71124555(12) GeV
$M_b$	4.78174390(33) GeV
$M_t$	173.720873(18) GeV
$y_t$	0.99780181(28) GeV
$M_u$	5.204992524(32) MeV
$M_d$	7.784107630(47) MeV
$M_s$	160.7159974(75) MeV

Table II. Ultra high precision Predicted pole masses and derived values assuming  $k^{12} = 3.5$  (exact).

- [11] C. Brannen, The Lepton Masses, <http://brannenworks.com/MASSES2.pdf>, (2006).
- [12] Yoshio Koide, Tribimaximal Neutrino Mixing and a Relation Between Neutrino- and Charged Lepton-Mass Spectra, *J.Phys.G*34:1653-1664,2007, doi:10.1088/0954-3899/34/7/006, arXiv:hep-ph/0605074.
- [13] Y. Koide, Charged Lepton Mass Relations in a Supersymmetric Yukawaon Model, *Phys.Rev.D*79:033009,2009, doi:10.1103/PhysRevD.79.033009, arXiv:0811.3470 [hep-ph].
- [14] Yukinari Sumino, Family Gauge Symmetry and Koide's Mass Formula, *Phys.Lett.B*671:477-480,2009, doi:10.1016/j.physletb.2008.12.060, arXiv:0812.2090 [hep-ph].
- [15] Y. Sumino, Family Gauge Symmetry as an Origin of Koide's Mass Formula and Charged Lepton Spectrum, *JHEP* 0905:075,2009, doi:10.1088/1126-6708/2009/05/075, arXiv:0812.2103 [hep-ph].
- [16] Y. Sumino, Family Gauge Symmetry as an Origin of Koide's Mass Formula and Charged Lepton Spectrum, arXiv:0903.3640 [hep-ph].
- [17] Yoshio Koide, Yukawaon Approach to the Sumino Relation for Charged Lepton Masses, *Phys.Lett.B*687:219-224,2010, doi:10.1016/j.physletb.2010.03.019, arXiv:1001.4877 [hep-ph].
- [18] A. Kartavtsev, A remark on the Koide relation for quarks, arXiv:1111.0480 [hep-ph].
- [19] Werner Rodejohann, He Zhang, Extended Empirical Fermion Mass Relation, *Phys.Lett.B*698:152-156,2011, doi:10.1016/j.physletb.2011.03.007, arXiv:1101.5525 [hep-ph].
- [20] Jerzy Kocik, The Koide Lepton Mass Formula and Geometry of Circles, arXiv:1201.2067 [physics.gen-ph].
- [21] Guan-Hua Gao, Nan Li, Explorations of two empirical formulas for fermion masses, *Eur. Phys. J. C* (2016) 76:140, doi:10.1140/epjc/s10052-016-3990-3.
- [22] Yoshio Koide, Hiroyuki Nishiura, Quark and Lepton Mass Matrix Model with Only Six Family-Independent Parameters, *Phys. Rev. D* 92, 111301 (2015), doi:10.1103/PhysRevD.92.111301, arXiv:1510.05370 [hep-ph].
- [23] Yoshio Koide, Hiroyuki Nishiura, Quark and Lepton Mass Matrices Described by Charged Lepton Masses, doi:10.1142/S021773231650125X, arXiv:1512.08386 [hep-ph].
- [24] Yoshio Koide, Hiroyuki Nishiura, Flavon VEV Scales in  $U(3) \times U(3)'$  Model, arXiv:1701.06287 [hep-ph].
- [25] Peter J. Mohr, David B. Newell, Barry N. Taylor, CODATA Recommended Values of the Fundamental Physical Constants: 2014, doi:10.1103/RevModPhys.88.035009, arXiv:1507.07956 [physics.atom-ph].
- [26] C. Patrignani et al. (Particle Data Group), *Chin. Phys. C*, 40, 100001 (2016).
- [27] M. H. Mac Gregor, Electron generation of leptons and hadrons with reciprocal  $\alpha$ -quantized lifetimes and masses, *Int.J.Mod.Phys. A*20 (2005) 719-798, doi:10.1142/S0217751X05021117, arXiv:hep-ph/0506033.
- [28] M. H. Mac Gregor, A "Muon Mass Tree" with alpha-quantized Lepton, Quark and Hadron Masses, arXiv:hep-ph/0607233
- [29] K. Hagiwara, R. Liao, A.D. Martin, Daisuke Nomura, T. Teubner,  $(g-2)_\mu$  and  $\alpha(M_Z^2)$  re-evaluated using new precise data, *J. Phys. G* 38 (2011) 085003, doi:10.1088/0954-3899/38/8/085003, arXiv:1105.3149 [hep-ph].
- [30] Ivan Esteban, M. C. Gonzalez-Garcia, Michele Maltoni, Ivan Martinez-Soler, Thomas Schwetz, Updated fit to three neutrino mixing: exploring the accelerator-reactor complementarity, IFT-UAM/CSIC-16-114, YITP-SB-16-45, arXiv:1611.01514 [hep-ph].
- [31] Elena Giusarma, Martina Gerbino, Olga Mena, Sunny Vagnozzi, Shirley Ho, Katherine Freese, On the improvement of cosmological neutrino mass bounds, *Phys. Rev. D* 94, 083522 (2016), doi:10.1103/PhysRevD.94.083522, arXiv:1605.04320 [astro-ph.CO].
- [32] Giorgio Cortiana, Top-quark mass measurements: review and perspectives, arXiv:1510.04483 [hep-ex].
- [33] G. Aad, *et al.* (ATLAS Collaboration), Determination of the top-quark pole mass using  $t\bar{t}+1$ -jet events collected with the ATLAS experiment in 7 TeV  $pp$  collisions, *JHEP* 10 (2015) 121, doi:10.1007/JHEP10(2015)121, arXiv:1507.01769 [hep-ex].
- [34] S. Aoki, *et al.*,  $1+1+1$  flavor QCD + QED simulation at the physical point, *PR D*86 034507, doi:10.1103/PhysRevD.86.034507, arXiv:1205.2961 [hep-lat].
- [35] R. G. H. Robertson, for the KATRIN Collaboration, *J.Phys.Conf.Ser.*120:052028,2008, doi:10.1088/1742-6596/120/5/052028, arXiv:0712.3893 [nucl-ex].